Doi: https://doi.og/10.1027/1015-5759/a000631

ESM 2. Mathematical formulations

ESM 2a. Global (cumulative) logits (or graded response model), originally proposed by Samejima (1969), see Bacci, Bartolucci & Gnaldi (2014), chap. 2:

$$g_x[\boldsymbol{\lambda}_j(\boldsymbol{\theta})] = \log \frac{\lambda^{(j)}_{x|\theta} + \cdots + \lambda^{(j)}_{l_j-1|\theta}}{\lambda^{(j)}_{0|\theta} + \cdots + \lambda^{(j)}_{x-1|\theta}} = \log \frac{p(X_j \geq x|\theta)}{p(X_j < x|\theta)}, \quad x = 1, \ldots, l_j - 1$$

and its multidimensional extension (Bacci, Bartolucci & Gnaldi 2014, chap. 3.1, Eq. 7):

$$\log \frac{p(X_j \geq x|\Theta = \boldsymbol{\theta})}{p(X_j < x|\Theta = \boldsymbol{\theta})} = \gamma_j(\sum_{d=1}^{s} \delta_{jd}\theta_d - \beta_{jx}), \quad x = 1, \ldots, l_j - 1$$

ESM 2b. Local (adjacent categories) logits (generalized partial credit model), originally proposed by Muraki (1992), see Bacci, Bartolucci & Gnaldi (2014), chap. 2:

$$g_x[\boldsymbol{\lambda}_j(\theta)] = \log \frac{\lambda^{(j)}_{x|\theta}}{\lambda^{(j)}_{x-1|\theta}} = \log \frac{p(X_j = x|\theta)}{p(X_j = x - 1|\theta)}, \quad x = 1, \ldots, l_j - 1$$

ESM 2c. continuation ratio logits (sequential model), originally proposed by Fienberg (1980), see Bacci, Bartolucci & Gnaldi (2014), chap. 2:

$$g_x[\boldsymbol{\lambda}_j(\theta)] = \log \frac{\lambda^{(j)}_{x+1|\theta} + \cdots + \lambda^{(j)}_{l_j-1|\theta}}{\lambda^{(j)}_{x|\theta}} = \log \frac{p(X_j > x|\theta)}{p(X_j = x|\theta)}, \quad x = 1, \ldots, l_j - 1$$