Supplementary Material for the Manuscript:

Continuous-Time Latent Markov Factor Analysis for Exploring Measurement Model Changes Across Time

Vogelsmeier, Leonie V.D.E [a], Vermunt, Jeroen K. [a], Böing-Messing, Florian [b,a], De Roover, Kim [a]

[a] Tilburg University, The Netherlands

[b] Jheronimus Academy of Data Science, The Netherlands

Supplement A

This supplement provides additional information on the convergence problems inherent to the phantom-variable approach of LMFA, which emerged from an additional simulation study that we conducted. In this extra simulation study, we used the same datasets as in discrete-time- (DT-) and continuous-time- (CT-) LMFA but we put the observations on a 1-hour grid and included the phantom variables. Note that, when missing data is part of the data matrix, the response probabilities $p\,(\mathbf{y}_{it}|\mathbf{s}_t)$ are changed to $p\,(\mathbf{y}_{it}|\mathbf{s}_t)^{\kappa_{it}}$, where $\kappa_{it} = 1$ if subject i provides information for time-point t and $\kappa_{it} = 0$ otherwise. While for $\kappa_{it} = 1$ nothing changes, for $\kappa_{it} = 0$, $p\,(\mathbf{y}_{it}|\mathbf{s}_t)^0 = 1$, so that the missing data do not influence the likelihood (Vermunt, Tran, & Magidson, 2008).

The overall simulation study results were very much comparable to CT-LMFA (which shows that the theoretical approximation works very well in practice) and are therefore not further discussed. However, while almost all analyses converged in DT-LMFA and CT-LMFA, 10.76 % of the replications in the phantom variable approach exhibited estimation problems, especially for the lowest level of the number of measurement occasions per day (i.e., $T_{day} = 3$). Closer investigation of the non-convergence problems revealed that they were caused by reaching the maximum number of EM iterations without convergence (despite the high number of 10,000 iterations). The problem is that fewer measurement occasions per day increase the amount of phantom variables in the dataset, which hampers convergence. Re-estimating the non-converged models with new starting values or increasing the number of iterations may help. However, it should be noted that also the computation time is influenced. To validly compare the computation times, we re-estimated the first replications for all conditions while allowing for up to 50,000 iterations in the phantom-variable approach to obtain the computation times when estimation is not interrupted by too few iterations. With an average of about 10 minutes, estimation in the phantom variable approach—on an i5 processor with 8GB RAM—took about three times longer for $T_{day} = 3$ than for $T_{day} = 6$ (Just to give a reference, the conditions with $T_{day} = 3$ took only about 2 minutes in CT-LMFA and 1 minute in DT-LMFA). Although this computation time is perfectly feasible, the phantom-variable approach can become infeasible for datasets with highly unequal time intervals and very fine grids (such as the application that was described in Section 4), which lead to very large numbers of empty rows with missing values only.

Moreover, we also observed that the percentage of local maxima amounted to 7.24 % for datasets analyzed with the phantom-variable approach, which is much higher than for the other two methods. Here, the local maxima especially occurred for the lowest level of the number of measurement occasions per day, $T_{day} = 3$ and hence again, just as it was the case for the convergence problems, the level with the most phantom variables in a dataset. More random start sets can reduce the probability of retaining local ML solutions (as briefly outlined in Section A.4).

Considering all the disadvantages of the phantom variable approach (i.e., cumbersome data-organization procedure, difficult decisions on the length of the time-interval, many required iterations and start sets when the number of phantom variables is large, and results that cannot be easily compared across studies), we advise against using the phantom variable approach, which is why we did not consider this approach in our main simulation study.

References Supplement A

Vermunt, J. K., Tran, B., & Magidson, J. (2008). Latent class models in longitudinal research. In *Handbook of Longitudinal Research: Design, Measurement, and Analysis* (pp. 373–385). Burlington, MA: Elsevier.

Supplement B

In the following, we provide the Latent GOLD syntax that we used to analyze our application data, more specifically, the syntax of the chosen model with two states and respectively two and three factors within the states.

```
model
title '17 [3 2]';
options
   algorithm
      tolerance=1e-008 emtolerance=1e-008 emiterations=6000 nriterations=0;
   startvalues
      seed=0 sets=100 tolerance=1e-005 iterations=100 PCA;
   bayes latent=1
categorical=1
poisson=1
variances=1 ;
montecarlo
   seed=0 replicates=500 tolerance=1e-008;
quadrature nodes=10;
   missing  includeall;
  output
      profile parameters standarderrors estimatedvalues classification probmeans iterationdetails
      WriteParameters = 'results_parameters17.csv'
      write = 'results17.csv'
      writeloadings='results_loadings17.txt';
  outfile
      'classification17.csv' classification;
variables
  caseid short_ID;
  timeinterval deltaT;
  dependent
  V1 continuous,
  V2 continuous,
  V3 continuous,
  V4 continuous,
  V5 continuous,
  V6 continuous,
  V7 continuous,
  V8 continuous,
  V9 continuous,
  V10 continuous,
  V11 continuous,
  V12 continuous,
  V13 continuous,
  V14 continuous,
  V15 continuous,
```

```
  V16 continuous,
  V17 continuous,
  V18 continuous,
  V19 continuous,
  V20 continuous;
 latent
   State nominal dynamic coding=first 2,
   F1 continuous dynamic,
   F2 continuous dynamic,
   F3 continuous dynamic;

   independent condition nominal;
equations
// factor variances
  (1) F1| State;
  (1) F2| State;
  (1) F3| State;

// Markov model
   State[=0] <- 1 ;
   State <- (~tra) 1 | State[-1] ;
//Dependent variables determined by state specific
V1      <- 1 | State + (a1)F1 | State + (b1)F2 | State + (c1)F3 | State;
V2      <- 1 | State + (a2)F1 | State + (b2)F2 | State + (c2)F3 | State;
V3      <- 1 | State + (a3)F1 | State + (b3)F2 | State + (c3)F3 | State;
V4      <- 1 | State + (a4)F1 | State + (b4)F2 | State + (c4)F3 | State;
V5      <- 1 | State + (a5)F1 | State + (b5)F2 | State + (c5)F3 | State;
V6      <- 1 | State + (a6)F1 | State + (b6)F2 | State + (c6)F3 | State;
V7      <- 1 | State + (a7)F1 | State + (b7)F2 | State + (c7)F3 | State;
V8      <- 1 | State + (a8)F1 | State + (b8)F2 | State + (c8)F3 | State;
V9      <- 1 | State + (a9)F1 | State + (b9)F2 | State + (c9)F3 | State;
V10     <- 1 | State + (a10)F1 | State + (b10)F2 | State + (c10)F3 | State;
V11     <- 1 | State + (a11)F1 | State + (b11)F2 | State + (c11)F3 | State;
V12     <- 1 | State + (a12)F1 | State + (b12)F2 | State + (c12)F3 | State;
V13     <- 1 | State + (a13)F1 | State + (b13)F2 | State + (c13)F3 | State;
V14     <- 1 | State + (a14)F1 | State + (b14)F2 | State + (c14)F3 | State;
V15     <- 1 | State + (a15)F1 | State + (b15)F2 | State + (c15)F3 | State;
V16     <- 1 | State + (a16)F1 | State + (b16)F2 | State + (c16)F3 | State;
V17     <- 1 | State + (a17)F1 | State + (b17)F2 | State + (c17)F3 | State;
V18     <- 1 | State + (a18)F1 | State + (b18)F2 | State + (c18)F3 | State;
V19     <- 1 | State + (a19)F1 | State + (b19)F2 | State + (c19)F3 | State;
V20     <- 1 | State + (a20)F1 | State + (b20)F2 | State + (c20)F3 | State;

//Variances

V1      | State;
V2      | State;
V3      | State;
V4      | State;
V5      | State;
V6      | State;
```

```
V7      | State;
V8      | State;
V9      | State;
V10     | State;
V11     | State;
V12     | State;
V13     | State;
V14     | State;
V15     | State;
V16     | State;
V17     | State;
V18     | State;
V19     | State;
V20     | State;

//constraints:
c1[2,] = 0;
c2[2,] = 0;
c3[2,] = 0;
c4[2,] = 0;
c5[2,] = 0;
c6[2,] = 0;
c7[2,] = 0;
c8[2,] = 0;
c9[2,] = 0;
c10[2,] = 0;
c11[2,] = 0;
c12[2,] = 0;
c13[2,] = 0;
c14[2,] = 0;
c15[2,] = 0;
c16[2,] = 0;
c17[2,] = 0;
c18[2,] = 0;
c19[2,] = 0;
c20[2,] = 0;


end model
```

Supplement C

In the following, we provide some additional information about the treatment and the Becks Depression Inventory (BDI; Beck, Rush, Shaw, & Emery, 1979) used in the presented application (Section 4). Regarding the treatment, all participants were randomly assigned to attend up to 20 sessions of either the cognitive behavior therapy (CBT; see Beck et al., 1979; $n = 60$) or the interpersonal psychotherapy (IPT; Klerman, Weissman, Rounsaville, & Chevron, 1984; $n = 62$). Note that there were also patients who were assigned to medication groups but that we focused on the therapy groups only. Furthermore, we did not distinguish between the two types of therapy to simplify the application, with the main purpose to simply demonstrate the use of CT-LMFA. For the requirements to participate, early termination reasons (e.g., dissatisfaction with treatment), and the explanation of the therapies and the procedure, you are referred to Elkin et al. (1989) where this has been extensively described.

With regard to the BDI measures, note that we removed the two items 'weight loss' and the dichotomous item whether this was 'wanted' from the original measurement because this distinction cannot be made in factor analysis. Since desired weight loss is not part of depression, we deemed it important to remove the item from our analyses.

References Supplement C

Beck, A. T., Rush, A. J., Shaw, B. F., & Emery, G. (1979). *Cognitive Therapy of Depression*. New York, NY: Guilford Press.

Elkin, I., Shea, M. T., Watkins, J. T., Imber, S. D., Sotsky, S. M., Collins, J. F., . . . Parloff, M. B. (1989). National Institute of Mental Health Treatment of Depression Collaborative Research Program. General effectiveness of treatments. *Archives of General Psychiatry, 46*, 971–982.

Klerman, G. L., Weissman, M. M., Rounsaville, B. J., & Chevron, E. S. (1984). *Interpersonal Psychtherapy of Depression*. New York, NY: Basic Books Inc Publishers.