Electronic Supplementary Material for *A Psychometric Evaluation of the Short Grit Scale: A Closer Look at its Factor Structure and Scale Functioning*

Items as Proprietary

Full items can be obtained in Duckworth and Quinn (2009). The items are listed as proprietary. Below we provide labels, subscales, and a keyword so that researchers can match the items listed in this article to the items listed in Duckworth and Quinn (2009) for the Grit-S.

Table A1. *Grit-S items and corresponding labels*

Label	Subscale	Item keyword(s)
GS1	CI	new ideas
GS2	PE	setbacks
GS3	CI	lost interest
GS4	PE	hard worker
GS5	CI	goal
GS6	CI	focus
GS7	PE	finish
GS8	PE	diligent

Note: CI=consistency in interest; PE=perseverance of effort

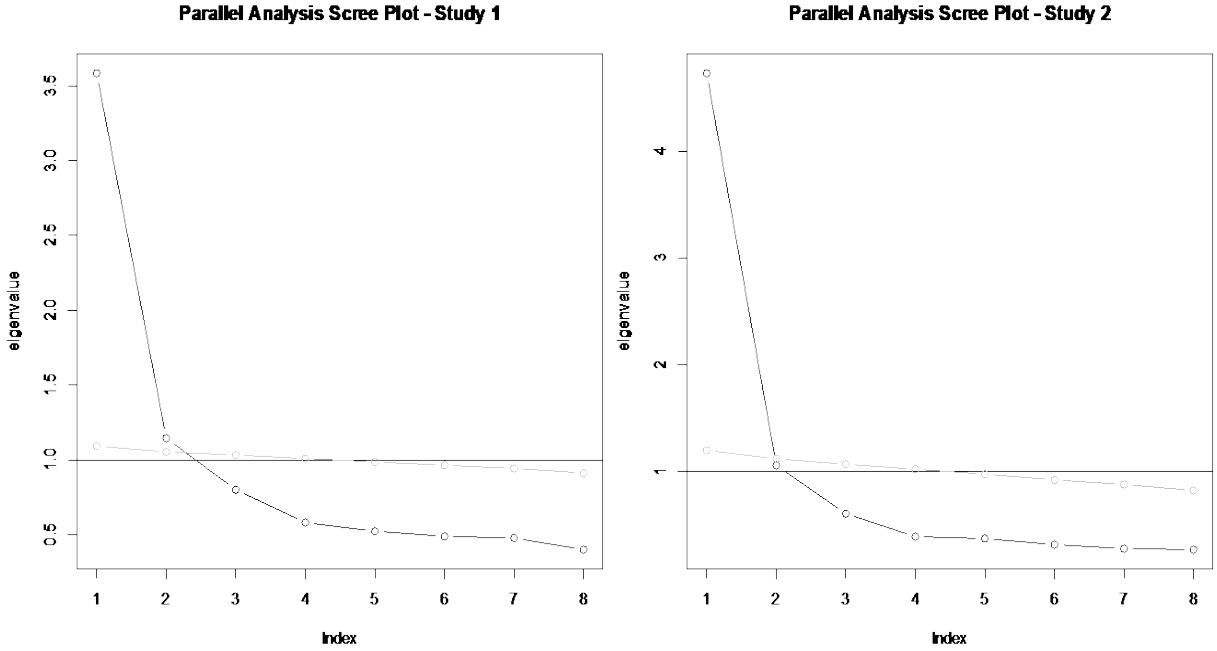

Figure A. Parallel Analysis Scree Plots for the Grit-S in Study 1 (left) and Study 2 (right). Note: vertical axes differ; gray lines= simulated eigenvalues; black lines=data eigenvalues

Table B. *Means, Standard Deviations, and Correlations Among the Grit-S Items*

Items	GS1	GS3	GS5	GS6	GS2	GS4	GS7	GS8
GS1	-	0.703	0.664	0.626	0.432	0.346	0.579	0.425
GS3	0.464	-	0.710	0.658	0.396	0.335	0.569	0.415
GS5	0.442	0.535	-	0.681	0.432	0.493	0.620	0.480
GS6	0.477	0.496	0.576	-	0.480	0.437	0.635	0.498
GS2	0.121	0.175	0.183	0.253	-	0.434	0.503	0.472
GS4	0.216	0.248	0.281	0.346	0.264	-	0.541	0.668
GS7	0.405	0.448	0.433	0.492	0.281	0.413	-	0.669
GS8	0.245	0.299	0.312	0.384	0.267	0.513	0.454	-
Mean S1	2.482	2.848	3.084	3.001	3.201	4.333	3.578	3.955
SD S1	1.128	1.272	1.183	1.292	1.176	0.901	1.062	0.955
Mean S2	3.102	3.088	3.059	4.291	3.331	3.473	3.741	4.069
SD S2	1.118	1.216	1.178	0.822	1.066	1.256	1.051	0.862

Note: correlations below the diagonal are correlations for items in Study 1 and correlations above the diagonal are correlations for items in Study 2. S1 = study 1; S2 = study 2; SD = standard deviation. Item labels are described in Table A.

Table C. *Unstandardized Factor Loadings and Intercepts for Partial Invariance Models across Gender, Education, and Race in Study 1.*

	Gender		Education					Race		
Factor λ	Females	Males	<H.S	H.S.	College	>College	Asian	Black	White	Other
GS1	.693			.701				.690		
GS3	.876			.862				.862		
GS5	.859			.838				.846		
GS6	1.003			.977				.985		
GS2	.381			.368				.376		
GS4	.405		.480	.430	.311	.241		.401		
GS7	.722			.696				.709		
GS8	.486			.444				.481		
GS4~GS8	.208	.305	.242	.244	.272	.175	.373	.043	.254	.236
κ	.000[a]	.074	-.097	0.000[a]	.358	.665	.091	.330	.000[a]	.041
ϕ	1.000[a]	.931	.856	1.000[a]	1.040	.838	.977	.925	1.000[a]	1.065
Intercepts										
GS1	2.464		2.342	2.497	2.298	2.139			2.452	
GS3	2.827				2.725				2.812	
GS5	3.062				2.959				3.055	
GS6	2.977				2.865				2.960	
GS2	3.188				3.152				3.182	
GS4	4.393	4.173	4.210	4.285	4.322	4.459	4.187	4.380	4.344	4.233
GS7	3.561				3.480				3.548	
GS8	4.003	3.821	3.791	3.846	3.982	4.055			3.939	

Note: λ = factor loading; ~ = correlated uniqueness; κ = latent variable mean; ϕ = latent variable variance; [a] = fixed for identification; <H.S. = less than high school education; H.S. high school education;

Description of the outcomes

The following are the descriptions of the self regulation measures from Eisenberg et al., (2018):

The Self-Regulation Questionnaire (SSRQ) is a 31-item self-report measure of the ability to regulate behavior to achieve one's goals. Participants indicate the extent to which they agree with each item using a 5-point Likert scale: 1 (Strongly Disagree), 2 (Somewhat Disagree), 3 (Neutral), 4 (Somewhat Agree), and 5 (Strongly Agree). The measure has one total scale computed by summing the items (after reverse-coding certain items, as needed). Examples of items include the following: "Once I have a goal, I can usually plan how to reach it," "I have a lot of willpower, " and "As soon as I see a problem or challenge, I start looking for possible solutions.

The Emotion Regulation Questionnaire (ERQ) is a 10-item self-report scale designed to assess habitual use of two commonly used strategies to alter emotion: cognitive reappraisal and expressive suppression. Participants respond to each item using a 7-point Likert scale ranging from 1 (strongly disagree) to 7 (strongly agree). Cognitive reappraisal involves thinking differently about a situation in order to change its meaning in order to alter one's emotional experience. Expressive suppression involves decreasing the outward expression of emotion. Six items contribute to the subscale for cognitive reappraisal (e.g., "When I'm faced with a stressful situation, I make myself think about it in a way that helps me stay calm"). Four items contribute to the subscale for expressive suppression (e.g., "When I am feeling negative emotions, I make sure not to express them")

The Mindful Attention Awareness Scale (MAAS) is a 15-item self-report survey that measures the tendency to be fully aware of one's experience in the present moment without distraction or forgetfulness. Participants indicate whether they frequently or infrequently experience each item using a 6-point Likert scale: 1 (Almost Always), 2 (Very Frequently), 3 (Somewhat Frequently), 4 (Somewhat Infrequently), 5 (Very Infrequently), and 6 (Almost Never). The scale was developed with the understanding that people likely have better conscious access to information about their tendency to be mindless rather than mindful. As a result, the total score for the MAAS is computed by reverse-scoring and then summing all items. Examples of items include the following: "I find it difficult to stay focused on what's happening in the present, " "I do jobs or tasks automatically, without being aware of what I'm doing, " and "I snack without being aware that I'm eating."

Item Fit

Below, we report the S-X^2 statistics to evaluate the fit of the IRT model to the dataset (Kang & Chen, 2008; Orlando & Thissen, 2000). These are chi-square tests, so the p-values should be above .05. Statistics suggest that the model fit the data well in Sample 1. In Sample 2, there were some problematic items, particularly item 4 and 8 which had correlated uniquenesses in the determined model and were also flagged for measurement bias.

Sample 1:

item		S_X2	df.S_X2	p.S_X2
1	GS1	78.818	66	0.134
2	GS2	65.584	67	0.526
3	GS3	72.618	64	0.215
4	GS4	73.426	60	0.114
5	GS5	91.634	83	0.242
6	GS6	67.993	64	0.343
7	GS7	74.499	64	0.174
8	GS8	68.897	63	0.285

Sample 2:

item		S_X2	df.S_X2	p.S_X2
1	GS1	50.895	41	0.138
2	GS2	68.236	57	0.146
3	GS3	57.617	41	0.044
4	GS4	57.539	38	0.022
5	GS5	37.945	34	0.294
6	GS6	31.634	39	0.793
7	GS7	42.731	32	0.097
8	GS8	73.696	38	0.000

Local Dependence Analysis.

Typically, when two items exhibit local dependence and one item from the pair is deleted, the factor loading of the other item typically decreases while the factor loadings for the rest of the items in the factor typically increase (Edwards et al., 2017). In this case we used the Jackknife Slope Index (JSI) to flag items that exhibit local dependence (Edwards et al., 2017). The goal of the JSI is to investigate how much factor loadings change when one of the items is removed from the scale, similar to a DFBETAS measure of influence from regression (Cohen, Cohen, West, & Aiken, 2003). High JSI values, defined as those greater than two standard deviations away from the mean of the JSI estimated values, indicate that factor loading estimates change substantially when one of the items is dropped.

Study 1. Our analysis did not flag any items for violations of local dependence with the suggested cutoff value. However, it is recommended to proceed with caution because the JSI value for the item pair GS4 and GS8 was -4.216, which is close to the JSI confidence limits of -4.714 and 4.797.

Study 2. The JSI did not flag any items for violations of local dependence with the suggested cutoff value but should be interpreted with caution. Specifically, the JSI value for the item pair GS4 and GS8 in Study 2 was -2.221 and the JSI confidence limits were -2.998 and 3.046.

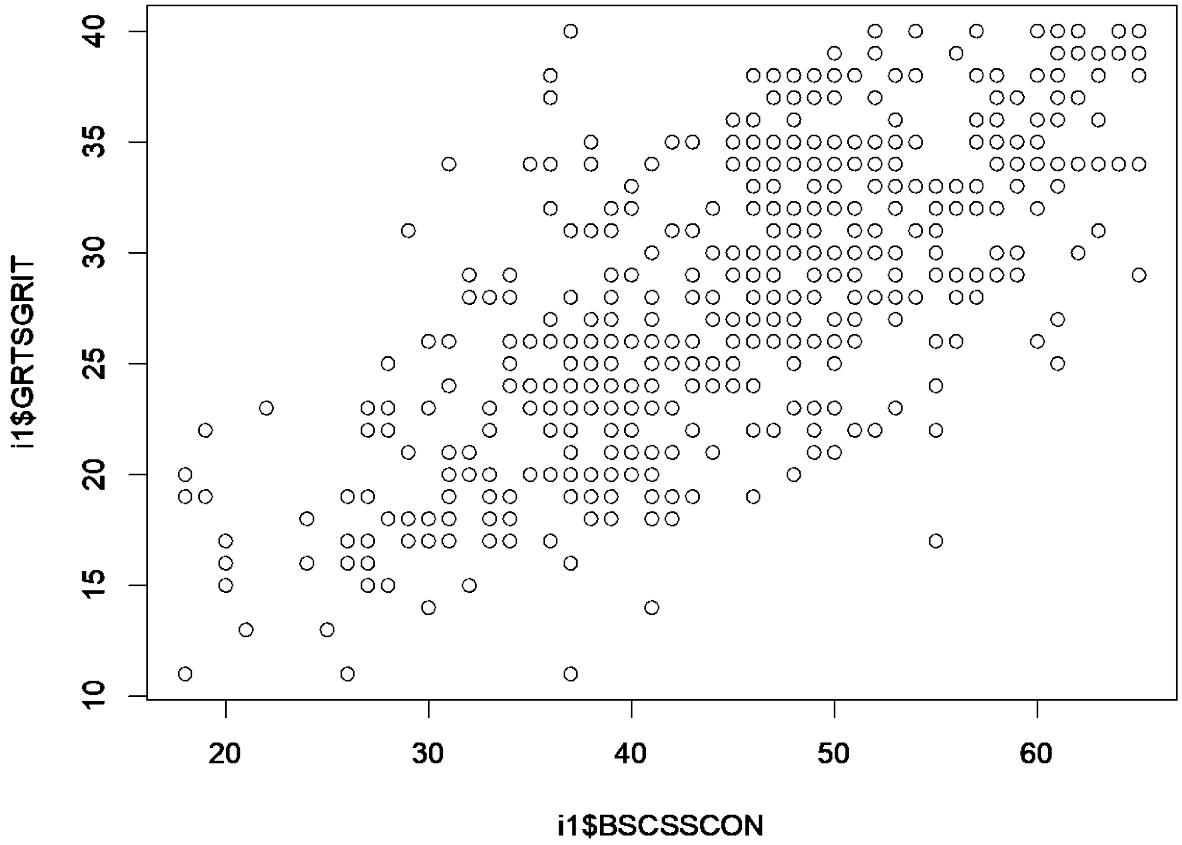

Figure B: Scatterplot for brief self-control questionnaire (x-axis) and girt (y-axis)

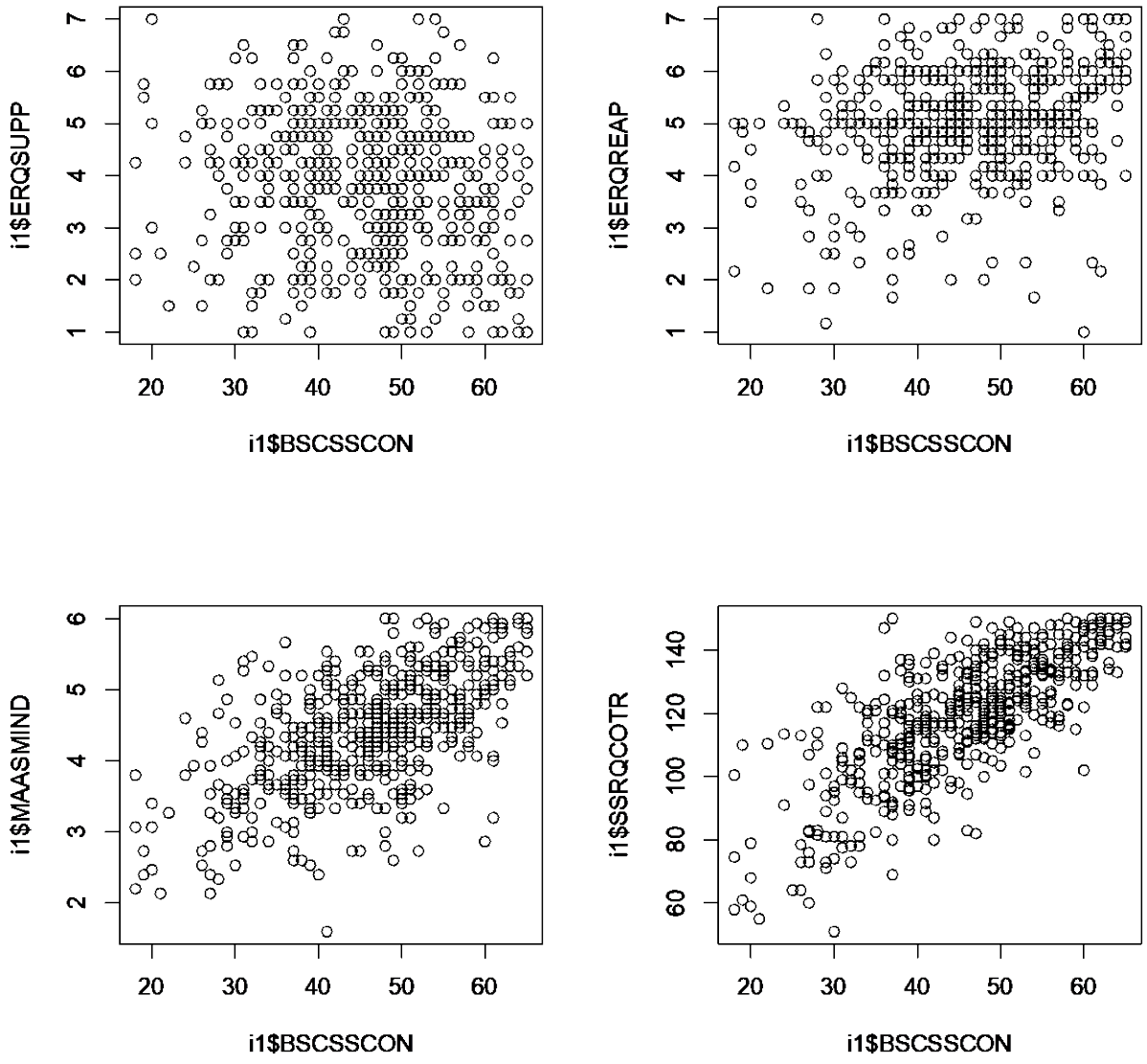

Figure C: Scatterplots for brief self-control and the extrinsic convergent validity outcomes – expressive suppression (top-left, ERQSUPP), cognitive reappraisal (top-right, ERQREAP), mindfulness (bottom-left, MAASMIND), and self-regulation (bottom-right ,SSRQCOTR).

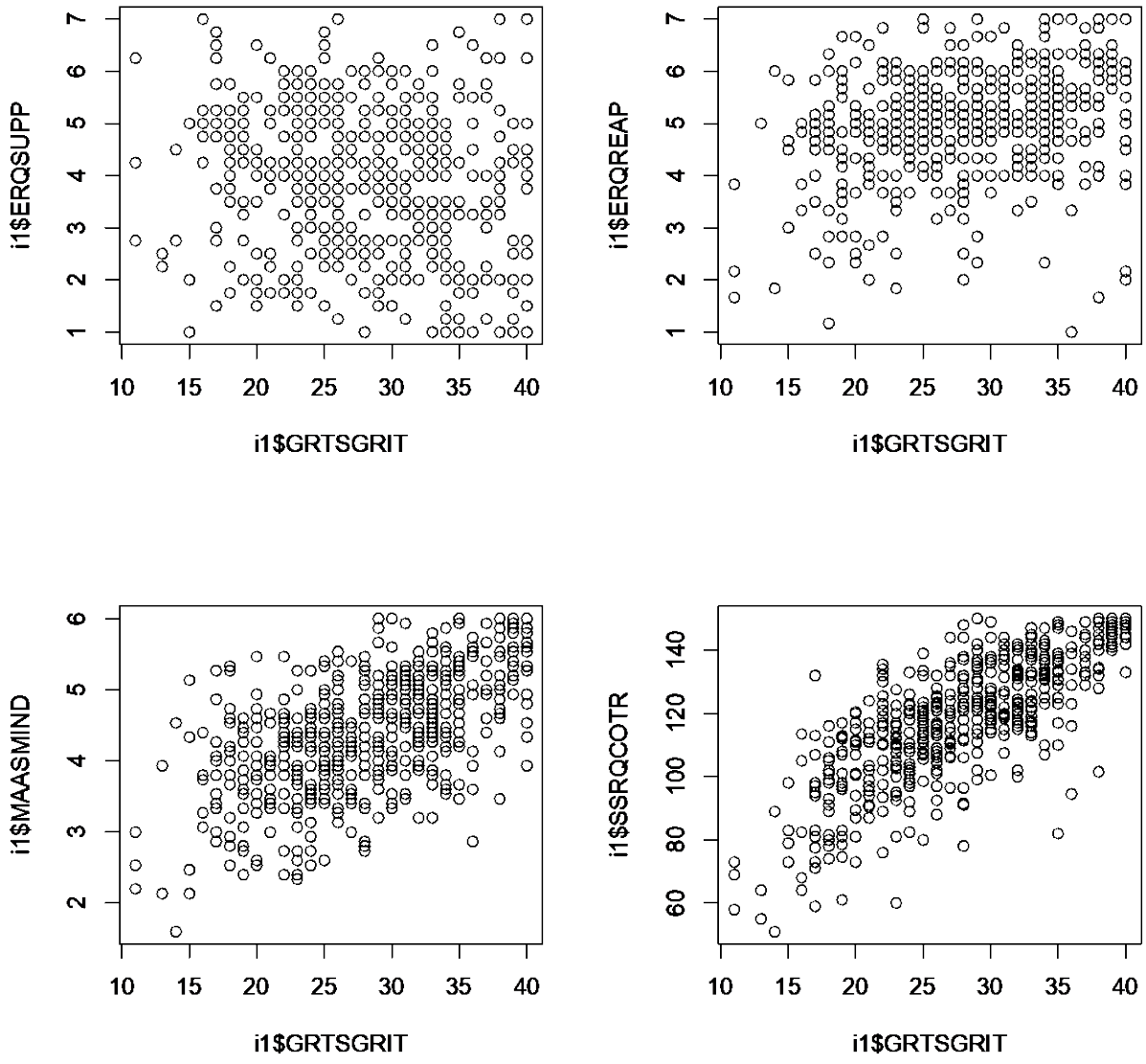

Figure D. Scatterplots for grit and the extrinsic convergent validity outcomes – expressive suppression (top-left, ERQSUPP), cognitive reappraisal (top-right, ERQREAP), mindfulness (bottom-left, MAASMIND), and self-regulation (bottom-right, SSRQCOTR).

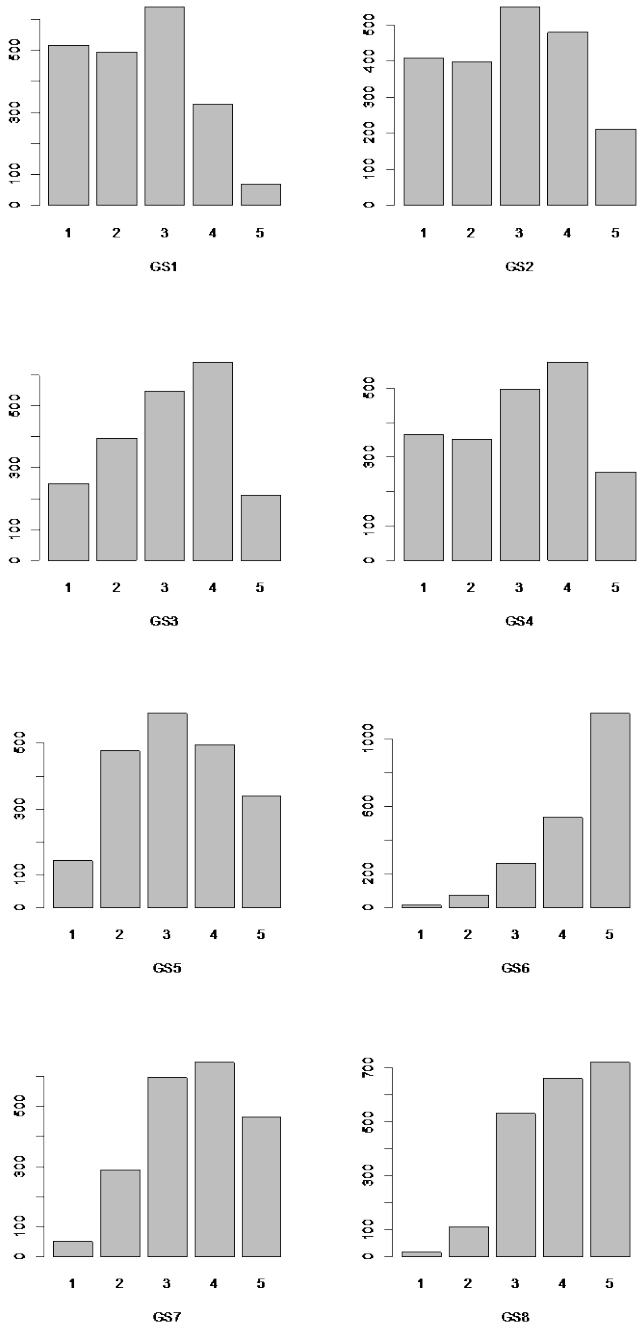

Figure E. Item frequency distributions for Sample 1

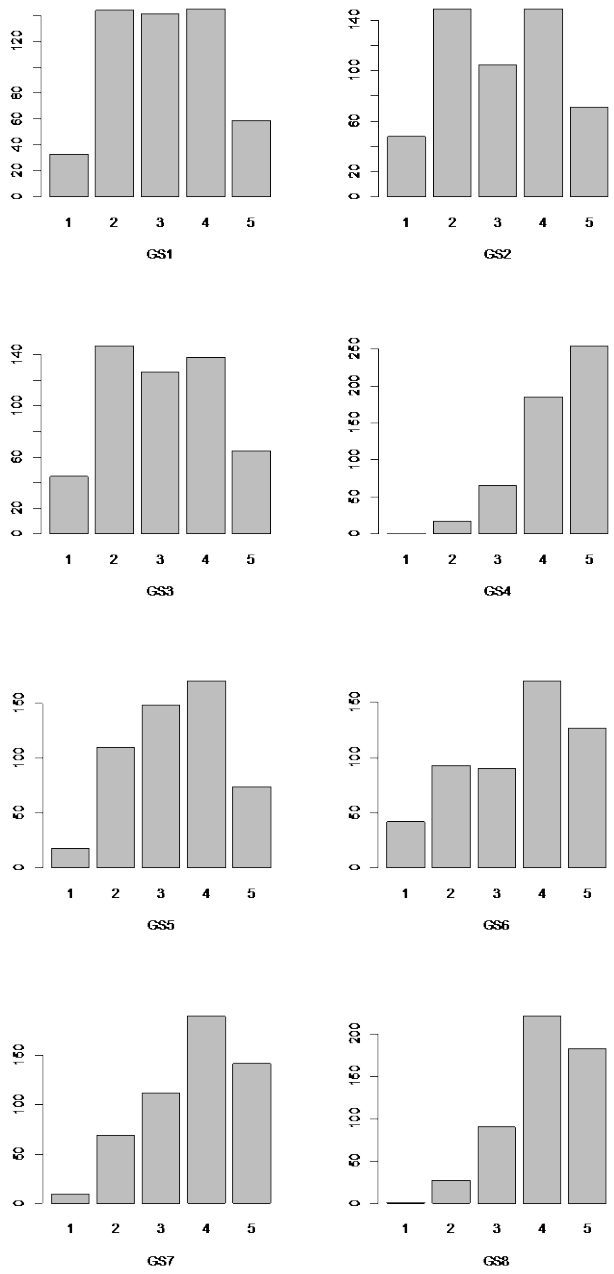

Figure F. Item frequency distributions for Sample 2.

References

Cohen, J., Cohen, P., West, S. G., & Aiken, L. S. (2003). *Applied multiple regression/correlation analysis for the behavioral sciences* (3rd ed.). Mahwah, NJ: Erlbaum.

Edwards, M. C., Houts, C. R., & Cai, L. (2018). A diagnostic procedure to detect departures from local independence in item response theory models. *Psychological Methods, 23*, 138-149

Kang, T., & Chen, T. T. (2008). Performance of the generalized S-X2 item fit index for polytomous IRT models. *Journal of Educational Measurement*, *45*(4), 391-406.

Orlando, M., & Thissen, D. (2000). Likelihood-based item-fit indices for dichotomous item response theory models. *Applied Psychological Measurement*, *24*(1), 50-64.

Rosseel, Y. (2012). lavaan: An R Package for Structural Equation Modeling. *Journal of Statistical Software*, *48*, 1-36

ESM: R inputs and outputs for analysis

```
##########
Sample 1
##########


#parallel analysis

library(lavaan)
gr2=read.csv('data.csv',header=TRUE)
l1=fa.parallel(gr2)
plot(l1$pc.values,type='b',ylab='eigenvalue',main='Parallel Analysis Scree Plot - Study 1')
abline(h=1)
lines(l1$pc.sim,type='b',col='gray')

# Output is Figure A, left panel


#Recoding data and fitting a bifactor model

library(lavaan)

gr3=reverse.code(c(1,1,1,1,-1,-1,-1,-1), gr2, mini = 1, maxi = 5)
model_b2='
f1=~GS2+GS5+GS7+GS8+GS4+GS6+GS9+GS12
g1=~GS2+GS5+GS7+GS8
g2=~GS4+GS6+GS9+GS12
'
colnames(gr3)=c('GS2','GS5','GS7','GS8','GS4','GS6','GS9','GS12')
mfit2=cfa(model_b2,gr3,std.lv=T,orthogonal=TRUE)
#summary(mfit2,standardized=TRUE)
fitMeasures(mfit2)
```

npar	fmin	chisq	df	pvalue	baseline.chisq
24.000	0.015	59.418	12.000	0.000	4848.626
baseline.df	baseline.pvalue	cfi	tli	nnfi	rfi
28.000	0.000	0.990	0.977	0.977	0.971
nfi	pnfi	ifi	rni	logl	unrestricted.logl
0.988	0.423	0.990	0.990	-22596.697	-22566.989
aic	bic	ntotal	bic2	rmsea	rmsea.ci.lower
45241.395	45376.374	2047.000	45300.124	0.044	0.033
rmsea.ci.upper	rmsea.pvalue	rmr	rmr_nomean	srmr	srmr_bentler
0.055	0.799	0.023	0.023	0.018	0.018
srmr_bentler_nomean	crmr	crmr_nomean	srmr_mplus	srmr_mplus_nomean	cn_05
0.018	0.020	0.020	0.018	0.018	725.372
cn_01	gfi	agfi	pgfi	mfi	ecvi
904.204	0.993	0.978	0.331	0.988	0.052

```
#Unidimensional Model

model_b3='
f1=~GS2+GS5+GS7+GS8+GS4+GS6+GS9+GS12
'
mfituni=cfa(model_b3,gr3,std.lv=T)
summary(mfituni,standardized=T)

lavaan 0.6-3 ended normally after 15 iterations

  Optimization method                           NLMINB
  Number of free parameters                         16

                                                  Used       Total
  Number of observations                          2047        2073

  Estimator                                         ML
  Model Fit Test Statistic                     568.957
  Degrees of freedom                                20
  P-value (Chi-square)                           0.000

Parameter Estimates:

  Information                                 Expected
  Information saturated (h1) model          Structured
  Standard Errors                             Standard

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  f1 =~
    GS2               0.673    0.024   27.491    0.000    0.673    0.596
    GS5               0.850    0.027   31.706    0.000    0.850    0.668
    GS7               0.833    0.025   33.881    0.000    0.833    0.703
    GS8               0.978    0.026   37.373    0.000    0.978    0.757
    GS4               0.390    0.027   14.248    0.000    0.390    0.332
    GS6               0.448    0.020   22.200    0.000    0.448    0.498
    GS9               0.729    0.022   32.804    0.000    0.729    0.686
    GS12              0.520    0.021   24.678    0.000    0.520    0.545
```

Variances:

	Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
.GS2	0.821	0.029	28.710	0.000	0.821	0.644
.GS5	0.895	0.033	27.170	0.000	0.895	0.553
.GS7	0.709	0.027	26.129	0.000	0.709	0.506
.GS8	0.714	0.030	23.957	0.000	0.714	0.428
.GS4	1.226	0.039	31.260	0.000	1.226	0.889
.GS6	0.611	0.020	30.043	0.000	0.611	0.752
.GS9	0.598	0.022	26.670	0.000	0.598	0.529
.GS12	0.640	0.022	29.485	0.000	0.640	0.703
f1	1.000			1.000	1.000	

```
#Unidimensional model with correlated residual

model_b4='
 f1=~GS2+GS5+GS7+GS8+GS4+GS6+GS9+GS12
 GS6~~GS12
 '
mfituni2=cfa(model_b4,gr3,std.lv=T)
summary(mfituni2,standardized=T)


lavaan 0.6-3 ended normally after 16 iterations

  Optimization method                           NLMINB
  Number of free parameters                         17

                                                  Used       Total
  Number of observations                          2047        2073

  Estimator                                         ML
  Model Fit Test Statistic                     296.306
  Degrees of freedom                                19
  P-value (Chi-square)                           0.000

Parameter Estimates:

  Information                                 Expected
  Information saturated (h1) model          Structured
  Standard Errors                             Standard

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  f1 =~
    GS2               0.687    0.024   28.113    0.000    0.687    0.609
    GS5               0.867    0.027   32.425    0.000    0.867    0.682
    GS7               0.850    0.024   34.708    0.000    0.850    0.718
    GS8               0.988    0.026   37.748    0.000    0.988    0.764
    GS4               0.375    0.028   13.641    0.000    0.375    0.320
    GS6               0.405    0.021   19.697    0.000    0.405    0.450
    GS9               0.714    0.022   31.846    0.000    0.714    0.672
    GS12              0.480    0.021   22.364    0.000    0.480    0.503

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  .GS6 ~~
    .GS12             0.247    0.017   14.663    0.000    0.247    0.371
```

Variances:

	Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
.GS2	0.802	0.028	28.387	0.000	0.802	0.630
.GS5	0.866	0.033	26.642	0.000	0.866	0.536
.GS7	0.680	0.027	25.406	0.000	0.680	0.485
.GS8	0.695	0.030	23.284	0.000	0.695	0.416
.GS4	1.238	0.040	31.303	0.000	1.238	0.898
.GS6	0.647	0.021	30.394	0.000	0.647	0.798
.GS9	0.619	0.023	26.916	0.000	0.619	0.548
.GS12	0.681	0.023	29.894	0.000	0.681	0.747
f1	1.000				1.000	1.000

#Two factor congeneric model

```
model_b5='
g1=~GS2+GS5+GS7+GS8
g2=~GS4+GS6+GS9+GS12
g1~~g2
'
mfit2=cfa(model_b5,gr3,std.lv=T)
summary(mfit2,standardized=T)
```

lavaan 0.6-3 ended normally after 17 iterations

Optimization method	NLMINB	
Number of free parameters	17	
	Used	Total
Number of observations	2047	2073
Estimator	ML	
Model Fit Test Statistic	226.350	
Degrees of freedom	19	
P-value (Chi-square)	0.000	

Parameter Estimates:

Information	Expected
Information saturated (h1) model	Structured
Standard Errors	Standard

Latent Variables:

	Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all

g1 =~						
GS2	0.697	0.025	28.395	0.000	0.697	0.618
GS5	0.879	0.027	32.703	0.000	0.879	0.691
GS7	0.876	0.025	35.689	0.000	0.876	0.739
GS8	0.998	0.026	37.783	0.000	0.998	0.772
g2 =~						
GS4	0.456	0.028	16.153	0.000	0.456	0.388
GS6	0.555	0.020	27.208	0.000	0.555	0.616
GS9	0.785	0.023	33.656	0.000	0.785	0.739
GS12	0.633	0.021	29.693	0.000	0.633	0.664

Covariances:

	Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
g1 ~~						
g2	0.746	0.017	42.637	0.000	0.746	0.746

Variances:

	Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
.GS2	0.788	0.028	27.883	0.000	0.788	0.619
.GS5	0.845	0.033	25.836	0.000	0.845	0.523
.GS7	0.636	0.027	23.807	0.000	0.636	0.453
.GS8	0.675	0.031	21.976	0.000	0.675	0.404
.GS4	1.171	0.039	30.397	0.000	1.171	0.849
.GS6	0.503	0.019	26.346	0.000	0.503	0.620
.GS9	0.513	0.025	20.805	0.000	0.513	0.455
.GS12	0.510	0.021	24.665	0.000	0.510	0.560
g1	1.000			1.000	1.000	
g2	1.000			1.000	1.000	

#Two factor noncongeneric model

fa(gr3,2)
Factor Analysis using method = minres
Call: fa(r = gr3, nfactors = 2)
Standardized loadings (pattern matrix) based upon correlation matrix

	MR1	MR2	h2	u2	com
GS2	0.67	-0.07	0.40	0.60	1.0
GS5	0.74	-0.04	0.51	0.49	1.0
GS7	0.75	-0.02	0.54	0.46	1.0
GS8	0.67	0.13	0.57	0.43	1.1
GS4	0.05	0.35	0.15	0.85	1.0
GS6	-0.03	0.72	0.49	0.51	1.0
GS9	0.40	0.39	0.50	0.50	2.0

GS12 0.02 0.71 0.53 0.47 1.0

```
#Partial Invariance Model for Gender

 p3_mod2<-cfa(metric,data=gr6,group="gender",
 group.equal=c("loadings","intercepts"),group.partial = c("GS6~1","GS12~1"),missing='ml')
 summary(p3_mod2,fit.measures=T)
```

lavaan 0.6-3 ended normally after 47 iterations

Optimization method	NLMINB
Number of free parameters	52
Number of equality constraints	14

Number of observations per group	
2	1387
1	660
Number of missing patterns per group	
2	1
1	1

Estimator	ML
Model Fit Test Statistic	358.674
Degrees of freedom	50
P-value (Chi-square)	0.000

Chi-square for each group:

2	217.651
1	141.023

Model test baseline model:

Minimum Function Test Statistic	4913.606
Degrees of freedom	56
P-value	0.000

User model versus baseline model:

Comparative Fit Index (CFI)	0.936
Tucker-Lewis Index (TLI)	0.929

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-22672.582
Loglikelihood unrestricted model (H1)	-22493.245

```
  Number of free parameters                         38
  Akaike (AIC)                               45421.164
  Bayesian (BIC)                             45634.881
  Sample-size adjusted Bayesian (BIC)        45514.152

Root Mean Square Error of Approximation:

  RMSEA                                          0.078
  90 Percent Confidence Interval          0.070  0.085
  P-value RMSEA <= 0.05                          0.000

Standardized Root Mean Square Residual:

  SRMR                                           0.052

Parameter Estimates:

  Information                                 Observed
  Observed information based on                Hessian
  Standard Errors                             Standard


Group 1 [2]:

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  f1 =~
    GS2     (.p1.)    0.693    0.026   26.505    0.000
    GS5     (.p2.)    0.876    0.029   29.869    0.000
    GS7     (.p3.)    0.859    0.027   31.681    0.000
    GS8     (.p4.)    1.003    0.029   34.368    0.000
    GS4     (.p5.)    0.381    0.028   13.489    0.000
    GS6     (.p6.)    0.405    0.021   19.275    0.000
    GS9     (.p7.)    0.722    0.024   29.525    0.000
    GS12    (.p8.)    0.486    0.023   21.589    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  .GS6 ~~
    .GS12             0.208    0.018   11.281    0.000

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
    .GS2    (.19.)    2.464    0.028   89.322    0.000
    .GS5    (.20.)    2.827    0.032   88.751    0.000
```

.GS7	(.21.)	3.062	0.030	101.932	0.000
.GS8	(.22.)	2.977	0.033	90.144	0.000
.GS4	(.23.)	3.188	0.027	118.780	0.000
.GS6		4.393	0.023	192.047	0.000
.GS9	(.25.)	3.561	0.026	134.681	0.000
.GS12		4.003	0.025	158.900	0.000
f1		0.000			

Variances:

	Estimate	Std.Err	z-value	P(>|z|)
f1	1.000			
.GS2	0.791	0.034	23.627	0.000
.GS5	0.869	0.039	22.207	0.000
.GS7	0.689	0.032	21.456	0.000
.GS8	0.626	0.033	18.688	0.000
.GS4	1.189	0.046	25.728	0.000
.GS6	0.562	0.022	24.979	0.000
.GS9	0.595	0.027	22.093	0.000
.GS12	0.644	0.026	24.482	0.000

Group 2 [1]:

Latent Variables:

		Estimate	Std.Err	z-value	P(>|z|)
f1 =~					
GS2	(.p1.)	0.693	0.026	26.505	0.000
GS5	(.p2.)	0.876	0.029	29.869	0.000
GS7	(.p3.)	0.859	0.027	31.681	0.000
GS8	(.p4.)	1.003	0.029	34.368	0.000
GS4	(.p5.)	0.381	0.028	13.489	0.000
GS6	(.p6.)	0.405	0.021	19.275	0.000
GS9	(.p7.)	0.722	0.024	29.525	0.000
GS12	(.p8.)	0.486	0.023	21.589	0.000

Covariances:

	Estimate	Std.Err	z-value	P(>|z|)
.GS6 ~~				
.GS12	0.305	0.034	8.884	0.000

Intercepts:

		Estimate	Std.Err	z-value	P(>|z|)
.GS2	(.19.)	2.464	0.028	89.322	0.000
.GS5	(.20.)	2.827	0.032	88.751	0.000
.GS7	(.21.)	3.062	0.030	101.932	0.000

.GS8	(.22.)	2.977	0.033	90.144	0.000
.GS4	(.23.)	3.188	0.027	118.780	0.000
.GS6		4.173	0.038	111.048	0.000
.GS9	(.25.)	3.561	0.026	134.681	0.000
.GS12		3.821	0.037	102.123	0.000
f1		0.074	0.051	1.457	0.145

Variances:

	Estimate	Std.Err	z-value	P(>|z|)
f1	0.931	0.075	12.403	0.000
.GS2	0.825	0.051	16.234	0.000
.GS5	0.860	0.056	15.225	0.000
.GS7	0.663	0.046	14.464	0.000
.GS8	0.833	0.058	14.365	0.000
.GS4	1.334	0.075	17.730	0.000
.GS6	0.802	0.046	17.390	0.000
.GS9	0.667	0.043	15.545	0.000
.GS12	0.737	0.043	17.113	0.000

```
#Partial Invariance Model for Education

config<-'
 f1=~c(NA,NA,NA,NA)*GS2+GS5+GS7+GS8+GS4+GS6+GS9+GS12
 f1~~c(1,NA,NA,NA)*f1
 f1~c(0,NA,NA,NA)*1
 GS12~~GS6
'
p2_mod3<-
cfa(config,data=gr6,group="education",missing="fiml",group.equal=c('loadings','intercepts'),group.par
tial=c('f1=~GS6','GS6~1','GS12~1','GS2~1'))


summary(p2_mod3,fit.measures=T,standardized=T)

lavaan 0.6-3 ended normally after 73 iterations

  Optimization method                           NLMINB
  Number of free parameters                        106
  Number of equality constraints                    36

  Number of observations per group
  2                                                 917
  1                                                 380
  3                                                 512
  4                                                 223
  Number of missing patterns per group
  2                                                   1
  1                                                   1
  3                                                   1
  4                                                   1

  Estimator                                        ML
  Model Fit Test Statistic                    420.421
  Degrees of freedom                              106
  P-value (Chi-square)                          0.000

Chi-square for each group:

  2                                           170.272
  1                                            81.509
  3                                           102.610
  4                                            66.031

Model test baseline model:
```

```
  Minimum Function Test Statistic             4666.955
  Degrees of freedom                           112
  P-value                                0.000

User model versus baseline model:

  Comparative Fit Index (CFI)                    0.931
  Tucker-Lewis Index (TLI)                       0.927

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)             -22422.175
  Loglikelihood unrestricted model (H1)     -22211.965

  Number of free parameters                        70
  Akaike (AIC)                           44984.351
  Bayesian (BIC)                          45377.525
  Sample-size adjusted Bayesian (BIC)        45155.130

Root Mean Square Error of Approximation:

  RMSEA                                     0.076
  90 Percent Confidence Interval          0.069  0.084
  P-value RMSEA <= 0.05                        0.000

Standardized Root Mean Square Residual:

  SRMR                                      0.058

Parameter Estimates:

  Information                            Observed
  Observed information based on               Hessian
  Standard Errors                          Standard


Group 1 [2]:

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  f1 =~
    GS2     (.p1.)  0.701    0.029   24.485    0.000    0.701    0.626
    GS5     (.p2.)  0.862    0.032   26.915    0.000    0.862    0.670
    GS7     (.p3.)  0.838    0.030   27.846    0.000    0.838    0.693
```

GS8	(.p4.)	0.977	0.033	29.691	0.000	0.977	0.760
GS4	(.p5.)	0.368	0.029	12.918	0.000	0.368	0.312
GS6		0.430	0.031	13.832	0.000	0.430	0.462
GS9	(.p7.)	0.696	0.027	25.766	0.000	0.696	0.656
GS12	(.p8.)	0.444	0.024	18.571	0.000	0.444	0.455

Covariances:

	Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
.GS6 ~~						
.GS12	0.244	0.027	9.073	0.000	0.244	0.340

Intercepts:

	Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
f1	0.000				0.000	0.000
.GS2	2.497	0.037	67.490	0.000	2.497	2.229
.GS5 (.21.)	2.725	0.037	73.899	0.000	2.725	2.119
.GS7 (.22.)	2.959	0.035	84.407	0.000	2.959	2.445
.GS8 (.23.)	2.865	0.039	73.039	0.000	2.865	2.229
.GS4 (.24.)	3.152	0.028	111.962	0.000	3.152	2.667
.GS6	4.285	0.031	139.318	0.000	4.285	4.601
.GS9 (.26.)	3.480	0.030	114.839	0.000	3.480	3.280
.GS12	3.846	0.032	119.295	0.000	3.846	3.939

Variances:

	Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
f1	1.000				1.000	1.000
.GS2	0.764	0.040	18.923	0.000	0.764	0.608
.GS5	0.911	0.050	18.228	0.000	0.911	0.551
.GS7	0.761	0.043	17.769	0.000	0.761	0.520
.GS8	0.699	0.044	15.933	0.000	0.699	0.423
.GS4	1.261	0.060	20.936	0.000	1.261	0.903
.GS6	0.683	0.034	20.129	0.000	0.683	0.787
.GS9	0.641	0.036	18.038	0.000	0.641	0.569
.GS12	0.756	0.037	20.259	0.000	0.756	0.793

Group 2 [1]:

Latent Variables:

		Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
f1 =~							
GS2	(.p1.)	0.701	0.029	24.485	0.000	0.649	0.578
GS5	(.p2.)	0.862	0.032	26.915	0.000	0.797	0.619
GS7	(.p3.)	0.838	0.030	27.846	0.000	0.776	0.673
GS8	(.p4.)	0.977	0.033	29.691	0.000	0.904	0.722

GS4	(.p5.)	0.368	0.029	12.918	0.000	0.341	0.308
GS6		0.480	0.052	9.146	0.000	0.444	0.467
GS9	(.p7.)	0.696	0.027	25.766	0.000	0.644	0.632
GS12	(.p8.)	0.444	0.024	18.571	0.000	0.411	0.459

Covariances:

	Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
.GS6 ~~						
.GS12	0.242	0.039	6.138	0.000	0.242	0.363

Intercepts:

	Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
f1	-0.097	0.066	-1.483	0.138	-0.105	-0.105
.GS2	2.342	0.057	41.273	0.000	2.342	2.087
.GS5 (.21.)	2.725	0.037	73.899	0.000	2.725	2.115
.GS7 (.22.)	2.959	0.035	84.407	0.000	2.959	2.569
.GS8 (.23.)	2.865	0.039	73.039	0.000	2.865	2.288
.GS4 (.24.)	3.152	0.028	111.962	0.000	3.152	2.849
.GS6	4.210	0.049	86.774	0.000	4.210	4.431
.GS9 (.26.)	3.480	0.030	114.839	0.000	3.480	3.416
.GS12	3.791	0.045	83.340	0.000	3.791	4.239

Variances:

	Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
f1	0.856	0.092	9.334	0.000	1.000	1.000
.GS2	0.838	0.068	12.380	0.000	0.838	0.666
.GS5	1.025	0.085	12.020	0.000	1.025	0.617
.GS7	0.725	0.063	11.457	0.000	0.725	0.547
.GS8	0.751	0.071	10.606	0.000	0.751	0.479
.GS4	1.108	0.082	13.468	0.000	1.108	0.905
.GS6	0.706	0.055	12.813	0.000	0.706	0.781
.GS9	0.623	0.053	11.854	0.000	0.623	0.600
.GS12	0.631	0.049	12.989	0.000	0.631	0.789

Group 3 [3]:

Latent Variables:

		Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
f1 =~							
GS2	(.p1.)	0.701	0.029	24.485	0.000	0.715	0.633
GS5	(.p2.)	0.862	0.032	26.915	0.000	0.879	0.718
GS7	(.p3.)	0.838	0.030	27.846	0.000	0.855	0.754
GS8	(.p4.)	0.977	0.033	29.691	0.000	0.996	0.764
GS4	(.p5.)	0.368	0.029	12.918	0.000	0.376	0.315

GS6		0.311	0.036	8.722	0.000	0.317	0.376
GS9	(.p7.)	0.696	0.027	25.766	0.000	0.710	0.661
GS12	(.p8.)	0.444	0.024	18.571	0.000	0.453	0.491

Covariances:

	Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
.GS6 ~~						
.GS12	0.272	0.032	8.549	0.000	0.272	0.433

Intercepts:

		Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
f1		0.358	0.063	5.696	0.000	0.351	0.351
.GS2		2.298	0.050	46.306	0.000	2.298	2.032
.GS5	(.21.)	2.725	0.037	73.899	0.000	2.725	2.228
.GS7	(.22.)	2.959	0.035	84.407	0.000	2.959	2.609
.GS8	(.23.)	2.865	0.039	73.039	0.000	2.865	2.198
.GS4	(.24.)	3.152	0.028	111.962	0.000	3.152	2.647
.GS6		4.322	0.039	110.784	0.000	4.322	5.125
.GS9	(.26.)	3.480	0.030	114.839	0.000	3.480	3.241
.GS12		3.982	0.041	97.247	0.000	3.982	4.317

Variances:

	Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
f1	1.040	0.097	10.752	0.000	1.000	1.000
.GS2	0.767	0.054	14.252	0.000	0.767	0.600
.GS5	0.724	0.056	13.048	0.000	0.724	0.484
.GS7	0.555	0.044	12.507	0.000	0.555	0.432
.GS8	0.707	0.057	12.433	0.000	0.707	0.416
.GS4	1.277	0.081	15.721	0.000	1.277	0.901
.GS6	0.611	0.039	15.527	0.000	0.611	0.859
.GS9	0.649	0.046	13.994	0.000	0.649	0.563
.GS12	0.646	0.043	15.151	0.000	0.646	0.759

Group 4 [4]:

Latent Variables:

		Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
f1 =~							
GS2	(.p1.)	0.701	0.029	24.485	0.000	0.642	0.579
GS5	(.p2.)	0.862	0.032	26.915	0.000	0.789	0.686
GS7	(.p3.)	0.838	0.030	27.846	0.000	0.767	0.715
GS8	(.p4.)	0.977	0.033	29.691	0.000	0.894	0.776
GS4	(.p5.)	0.368	0.029	12.918	0.000	0.337	0.284
GS6		0.241	0.048	4.991	0.000	0.220	0.328

GS9	(.p7.)	0.696	0.027	25.766	0.000	0.637	0.658
GS12	(.p8.)	0.444	0.024	18.571	0.000	0.406	0.491

Covariances:

	Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
.GS6 ~~						
.GS12	0.175	0.034	5.101	0.000	0.175	0.383

Intercepts:

	Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
f1	0.665	0.080	8.323	0.000	0.727	0.727
.GS2	2.139	0.071	30.148	0.000	2.139	1.930
.GS5 (.21.)	2.725	0.037	73.899	0.000	2.725	2.371
.GS7 (.22.)	2.959	0.035	84.407	0.000	2.959	2.757
.GS8 (.23.)	2.865	0.039	73.039	0.000	2.865	2.487
.GS4 (.24.)	3.152	0.028	111.962	0.000	3.152	2.659
.GS6	4.459	0.054	82.094	0.000	4.459	6.631
.GS9 (.26.)	3.480	0.030	114.839	0.000	3.480	3.592
.GS12	4.055	0.055	74.247	0.000	4.055	4.898

Variances:

	Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
f1	0.838	0.107	7.863	0.000	1.000	1.000
.GS2	0.816	0.085	9.597	0.000	0.816	0.664
.GS5	0.700	0.079	8.909	0.000	0.700	0.529
.GS7	0.563	0.065	8.633	0.000	0.563	0.489
.GS8	0.528	0.068	7.716	0.000	0.528	0.398
.GS4	1.292	0.124	10.402	0.000	1.292	0.919
.GS6	0.404	0.039	10.331	0.000	0.404	0.893
.GS9	0.532	0.058	9.155	0.000	0.532	0.567
.GS12	0.520	0.052	9.996	0.000	0.520	0.759

```
#Partial Invariance Model for Race


p3_mod<-
cfa(config,data=gr7,group="race",missing="fiml",group.equal=c('loadings','intercepts'),group.partial='
GS4~1')

Warning message:
In lav_data_full(data = data, group = group, cluster = cluster,  :
  lavaan WARNING: group variable 'race' contains missing values

summary(p3_mod,fit.measures=T,standardized=T)

lavaan 0.6-3 ended normally after 77 iterations

  Optimization method                       NLMINB
  Number of free parameters                    106
  Number of equality constraints                45

  Number of observations per group
  4                                           1435
  1                                            168
  3                                            157
  5                                            265
  Number of missing patterns per group
  4                                              1
  1                                              1
  3                                              1
  5                                              1

  Estimator                                     ML
  Model Fit Test Statistic                 429.963
  Degrees of freedom                           115
  P-value (Chi-square)                       0.000

Chi-square for each group:

  4                                        199.936
  1                                         76.136
  3                                         66.333
  5                                         87.558

Model test baseline model:

  Minimum Function Test Statistic          4919.335
```

Degrees of freedom	112
P-value	0.000

User model versus baseline model:

Comparative Fit Index (CFI)	0.934
Tucker-Lewis Index (TLI)	0.936

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-22415.352
Loglikelihood unrestricted model (H1)	-22200.370
Number of free parameters	61
Akaike (AIC)	44952.703
Bayesian (BIC)	45295.116
Sample-size adjusted Bayesian (BIC)	45101.315

Root Mean Square Error of Approximation:

RMSEA	0.074
90 Percent Confidence Interval	0.066 0.081
P-value RMSEA <= 0.05	0.000

Standardized Root Mean Square Residual:

SRMR	0.056

Parameter Estimates:

Information	Observed
Observed information based on	Hessian
Standard Errors	Standard

Group 1 [4]:

Latent Variables:

		Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
f1 =~							
GS2	(.p1.)	0.690	0.026	26.953	0.000	0.690	0.627
GS5	(.p2.)	0.862	0.029	29.996	0.000	0.862	0.687
GS7	(.p3.)	0.846	0.027	31.917	0.000	0.846	0.730
GS8	(.p4.)	0.985	0.029	34.106	0.000	0.985	0.759
GS4	(.p5.)	0.377	0.028	13.378	0.000	0.377	0.325

GS6	(.p6.)	0.401	0.021	19.096	0.000	0.401	0.456
GS9	(.p7.)	0.709	0.024	29.454	0.000	0.709	0.673
GS12	(.p8.)	0.481	0.022	21.603	0.000	0.481	0.507

Covariances:

	Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
.GS6 ~~						
.GS12	0.255	0.020	13.017	0.000	0.255	0.398

Intercepts:

		Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
f1		0.000			0.000	0.000	
.GS2	(.20.)	2.453	0.027	90.329	0.000	2.453	2.227
.GS5	(.21.)	2.812	0.031	90.243	0.000	2.812	2.242
.GS7	(.22.)	3.055	0.029	104.647	0.000	3.055	2.637
.GS8	(.23.)	2.961	0.033	90.719	0.000	2.961	2.283
.GS4		3.181	0.031	104.011	0.000	3.181	2.746
.GS6	(.25.)	4.327	0.021	207.692	0.000	4.327	4.920
.GS9	(.26.)	3.548	0.026	136.578	0.000	3.548	3.370
.GS12	(.27.)	3.943	0.022	175.546	0.000	3.943	4.156

Variances:

	Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
f1	1.000			1.000	1.000	
.GS2	0.736	0.031	23.640	0.000	0.736	0.607
.GS5	0.830	0.037	22.352	0.000	0.830	0.528
.GS7	0.627	0.030	21.036	0.000	0.627	0.467
.GS8	0.712	0.035	20.204	0.000	0.712	0.423
.GS4	1.201	0.046	26.207	0.000	1.201	0.894
.GS6	0.613	0.024	25.488	0.000	0.613	0.792
.GS9	0.606	0.027	22.588	0.000	0.606	0.546
.GS12	0.669	0.027	24.985	0.000	0.669	0.743

Group 2 [1]:

Latent Variables:

		Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
f1 =~							
GS2	(.p1.)	0.690	0.026	26.953	0.000	0.682	0.581
GS5	(.p2.)	0.862	0.029	29.996	0.000	0.851	0.653
GS7	(.p3.)	0.846	0.027	31.917	0.000	0.835	0.706
GS8	(.p4.)	0.985	0.029	34.106	0.000	0.972	0.755
GS4	(.p5.)	0.377	0.028	13.378	0.000	0.372	0.315
GS6	(.p6.)	0.401	0.021	19.096	0.000	0.396	0.396

GS9	(.p7.)	0.709	0.024	29.454	0.000	0.700	0.654
GS12	(.p8.)	0.481	0.022	21.603	0.000	0.475	0.498

Covariances:

	Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
.GS6 ~~						
.GS12	0.395	0.071	5.601	0.000	0.395	0.519

Intercepts:

		Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
f1		0.090	0.089	1.015	0.310	0.091	0.091
.GS2	(.20.)	2.453	0.027	90.329	0.000	2.453	2.091
.GS5	(.21.)	2.812	0.031	90.243	0.000	2.812	2.157
.GS7	(.22.)	3.055	0.029	104.647	0.000	3.055	2.583
.GS8	(.23.)	2.961	0.033	90.719	0.000	2.961	2.299
.GS4		3.073	0.088	34.856	0.000	3.073	2.604
.GS6	(.25.)	4.327	0.021	207.692	0.000	4.327	4.321
.GS9	(.26.)	3.548	0.026	136.578	0.000	3.548	3.316
.GS12	(.27.)	3.943	0.022	175.546	0.000	3.943	4.130

Variances:

	Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
f1	0.975	0.136	7.154	0.000	1.000	1.000
.GS2	0.911	0.110	8.311	0.000	0.911	0.662
.GS5	0.976	0.124	7.887	0.000	0.976	0.574
.GS7	0.702	0.094	7.466	0.000	0.702	0.502
.GS8	0.714	0.102	6.976	0.000	0.714	0.430
.GS4	1.255	0.140	8.984	0.000	1.255	0.901
.GS6	0.846	0.096	8.793	0.000	0.846	0.843
.GS9	0.655	0.083	7.874	0.000	0.655	0.572
.GS12	0.686	0.080	8.575	0.000	0.686	0.752

Group 3 [3]:

Latent Variables:

		Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
f1 =~							
GS2	(.p1.)	0.690	0.026	26.953	0.000	0.664	0.567
GS5	(.p2.)	0.862	0.029	29.996	0.000	0.829	0.635
GS7	(.p3.)	0.846	0.027	31.917	0.000	0.813	0.673
GS8	(.p4.)	0.985	0.029	34.106	0.000	0.947	0.755
GS4	(.p5.)	0.377	0.028	13.378	0.000	0.362	0.284
GS6	(.p6.)	0.401	0.021	19.096	0.000	0.386	0.494
GS9	(.p7.)	0.709	0.024	29.454	0.000	0.682	0.675

GS12	(.p8.)	0.481	0.022	21.603	0.000	0.463	0.488

Covariances:

		Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
.GS6 ~~							
.GS12		0.045	0.048	0.938	0.348	0.045	0.081

Intercepts:

		Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
f1		0.336	0.090	3.736	0.000	0.349	0.349
.GS2	(.20.)	2.453	0.027	90.329	0.000	2.453	2.097
.GS5	(.21.)	2.812	0.031	90.243	0.000	2.812	2.154
.GS7	(.22.)	3.055	0.029	104.647	0.000	3.055	2.527
.GS8	(.23.)	2.961	0.033	90.719	0.000	2.961	2.362
.GS4		3.198	0.099	32.187	0.000	3.198	2.513
.GS6	(.25.)	4.327	0.021	207.692	0.000	4.327	5.539
.GS9	(.26.)	3.548	0.026	136.578	0.000	3.548	3.514
.GS12	(.27.)	3.943	0.022	175.546	0.000	3.943	4.159

Variances:

		Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
f1		0.924	0.133	6.946	0.000	1.000	1.000
.GS2		0.928	0.115	8.062	0.000	0.928	0.678
.GS5		1.018	0.132	7.743	0.000	1.018	0.597
.GS7		0.801	0.107	7.499	0.000	0.801	0.548
.GS8		0.675	0.101	6.695	0.000	0.675	0.430
.GS4		1.489	0.171	8.723	0.000	1.489	0.919
.GS6		0.462	0.056	8.253	0.000	0.462	0.756
.GS9		0.555	0.074	7.483	0.000	0.555	0.544
.GS12		0.685	0.082	8.339	0.000	0.685	0.762

Group 4 [5]:

Latent Variables:

		Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
f1 =~							
GS2	(.p1.)	0.690	0.026	26.953	0.000	0.712	0.589
GS5	(.p2.)	0.862	0.029	29.996	0.000	0.890	0.679
GS7	(.p3.)	0.846	0.027	31.917	0.000	0.873	0.670
GS8	(.p4.)	0.985	0.029	34.106	0.000	1.016	0.795
GS4	(.p5.)	0.377	0.028	13.378	0.000	0.389	0.325
GS6	(.p6.)	0.401	0.021	19.096	0.000	0.414	0.415
GS9	(.p7.)	0.709	0.024	29.454	0.000	0.732	0.658
GS12	(.p8.)	0.481	0.022	21.603	0.000	0.497	0.511

Covariances:

	Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
.GS6 ~~						
.GS12	0.242	0.052	4.641	0.000	0.242	0.319

Intercepts:

		Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
f1		0.029	0.075	0.392	0.695	0.029	0.029
.GS2	(.20.)	2.453	0.027	90.329	0.000	2.453	2.028
.GS5	(.21.)	2.812	0.031	90.243	0.000	2.812	2.145
.GS7	(.22.)	3.055	0.029	104.647	0.000	3.055	2.347
.GS8	(.23.)	2.961	0.033	90.719	0.000	2.961	2.316
.GS4		3.249	0.071	45.660	0.000	3.249	2.716
.GS6	(.25.)	4.327	0.021	207.692	0.000	4.327	4.340
.GS9	(.26.)	3.548	0.026	136.578	0.000	3.548	3.193
.GS12	(.27.)	3.943	0.022	175.546	0.000	3.943	4.060

Variances:

	Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
f1	1.065	0.121	8.808	0.000	1.000	1.000
.GS2	0.954	0.091	10.503	0.000	0.954	0.653
.GS5	0.927	0.094	9.825	0.000	0.927	0.539
.GS7	0.933	0.094	9.928	0.000	0.933	0.551
.GS8	0.602	0.075	8.042	0.000	0.602	0.368
.GS4	1.281	0.114	11.239	0.000	1.281	0.895
.GS6	0.823	0.074	11.059	0.000	0.823	0.828
.GS9	0.700	0.070	9.933	0.000	0.700	0.567
.GS12	0.697	0.065	10.779	0.000	0.697	0.739

```
#IRT item parameters

n5=mirt(gr3,1)
Iteration: 25, Log-Lik: -21285.496, Max-Change: 0.00008

coef(n5,IRTpars=TRUE)
$GS2
      a     b1     b2    b3    b4
par 1.448 -1.037 -0.038 1.325 2.956

$GS5
      a     b1     b2    b3    b4
par 1.741 -1.188 -0.376 0.592 1.791

$GS7
      a     b1     b2    b3    b4
par 1.937 -1.578 -0.655 0.284 1.706

$GS8
      a     b1     b2    b3    b4
par 2.315 -1.158 -0.481 0.299 1.438

$GS4
      a     b1     b2    b3    b4
par 0.684 -4.075 -1.361 0.573 2.556

$GS6
      a     b1     b2     b3     b4
par 1.219 -4.461 -2.986 -1.582 -0.253

$GS9
      a     b1     b2     b3    b4
par 1.867 -2.744 -1.318 -0.125 1.032

$GS12
      a     b1    b2     b3    b4
par 1.279 -4.304 -2.59 -0.735 0.632

$GroupPars
   MEAN_1 COV_11
par      0      1
```

```
##########
Sample 2
##########

ont2=read.csv('data2.csv',header=TRUE)

#Parallel analysis plot
l1=fa.parallel(ont2)
plot(l1$pc.values,type='b',ylab='eigenvalue',main='Parallel Analysis Scree Plot - Study 2')
abline(h=1)
lines(l1$pc.sim,type='b',col='gray')

#see Figure A


##Bifactor model for Factor Strength


m3='
 f1=~GS1+GS3+GS5+GS6+GS2+GS4+GS7+GS8
 g1=~GS1+GS3+GS5+GS6
 g2=~GS2+GS4+GS7+GS8
'
 fitm3=cfa(m3,data=ont2,std.lv=T,orthogonal=T)
 summary(fitm3,fit.measures=T,standardized=T)


lavaan 0.6-3 ended normally after 33 iterations

  Optimization method                           NLMINB
  Number of free parameters                         24

  Number of observations                           522

  Estimator                                         ML
  Model Fit Test Statistic                      11.356
  Degrees of freedom                                12
  P-value (Chi-square)                           0.499

Model test baseline model:

  Minimum Function Test Statistic             2411.252
  Degrees of freedom                                28
  P-value                                        0.000
```

User model versus baseline model:

Comparative Fit Index (CFI)	1.000
Tucker-Lewis Index (TLI)	1.001

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-4966.132
Loglikelihood unrestricted model (H1)	-4960.454
Number of free parameters	24
Akaike (AIC)	9980.263
Bayesian (BIC)	10082.447
Sample-size adjusted Bayesian (BIC)	10006.266

Root Mean Square Error of Approximation:

RMSEA	0.000
90 Percent Confidence Interval	0.000 0.043
P-value RMSEA <= 0.05	0.982

Standardized Root Mean Square Residual:

SRMR	0.012

Parameter Estimates:

Information	Expected
Information saturated (h1) model	Structured
Standard Errors	Standard

Latent Variables:

	Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
f1 =~						
GS1	0.814	0.054	15.024	0.000	0.814	0.729
GS3	0.843	0.059	14.197	0.000	0.843	0.716
GS5	0.840	0.048	17.414	0.000	0.840	0.789
GS6	1.024	0.056	18.430	0.000	1.024	0.816
GS2	0.696	0.055	12.568	0.000	0.696	0.573
GS4	0.418	0.040	10.574	0.000	0.418	0.509
GS7	0.823	0.047	17.337	0.000	0.823	0.783
GS8	0.520	0.041	12.724	0.000	0.520	0.603
g1 =~						
GS1	0.363	0.101	3.577	0.000	0.363	0.325
GS3	0.657	0.093	7.023	0.000	0.657	0.558

```
    GS5       0.281    0.090    3.119    0.002    0.281    0.264
    GS6       0.162    0.107    1.509    0.131    0.162    0.129
  g2 =~
    GS2       0.244    0.066    3.724    0.000    0.244    0.201
    GS4       0.438    0.048    9.212    0.000    0.438    0.534
    GS7       0.300    0.057    5.267    0.000    0.300    0.286
    GS8       0.583    0.048   12.198    0.000    0.583    0.677

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  f1 ~~
    g1                0.000                             0.000    0.000
    g2                0.000                             0.000    0.000
  g1 ~~
    g2                0.000                             0.000    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .GS1               0.453    0.039   11.656    0.000    0.453    0.363
   .GS3               0.244    0.117    2.084    0.037    0.244    0.176
   .GS5               0.349    0.028   12.423    0.000    0.349    0.308
   .GS6               0.500    0.050    9.925    0.000    0.500    0.318
   .GS2               0.931    0.062   15.087    0.000    0.931    0.631
   .GS4               0.307    0.031   10.028    0.000    0.307    0.456
   .GS7               0.336    0.031   10.733    0.000    0.336    0.304
   .GS8               0.132    0.044    3.025    0.002    0.132    0.178
    f1                1.000                             1.000    1.000
    g1                1.000                             1.000    1.000
    g2                1.000                             1.000    1.000
```

Unidimensional Model

```
 m1='
 f1=~GS1+GS2+GS3+GS4+GS5+GS6+GS7+GS8
 '
 fitm1=cfa(m1,data=ont2,std.lv=T)
 summary(fitm1,fit.measures=T,standardized=T)
```

```
lavaan 0.6-3 ended normally after 17 iterations
```

Optimization method	NLMINB
Number of free parameters	16
Number of observations	522
Estimator	ML
Model Fit Test Statistic	329.675
Degrees of freedom	20
P-value (Chi-square)	0.000

Model test baseline model:

Minimum Function Test Statistic	2411.252
Degrees of freedom	28
P-value	0.000

User model versus baseline model:

Comparative Fit Index (CFI)	0.870
Tucker-Lewis Index (TLI)	0.818

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-5125.291
Loglikelihood unrestricted model (H1)	-4960.454
Number of free parameters	16
Akaike (AIC)	10282.583
Bayesian (BIC)	10350.705
Sample-size adjusted Bayesian (BIC)	10299.918

Root Mean Square Error of Approximation:

RMSEA	0.172
90 Percent Confidence Interval	0.156 0.189
P-value RMSEA <= 0.05	0.000

Standardized Root Mean Square Residual:

SRMR	0.077

Parameter Estimates:

Information	Expected
Information saturated (h1) model	Structured

Standard Errors Standard

Latent Variables:

	Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
f1 =~						
GS1	0.864	0.042	20.395	0.000	0.864	0.773
GS2	0.710	0.050	14.098	0.000	0.710	0.584
GS3	0.926	0.044	20.932	0.000	0.926	0.787
GS4	0.464	0.034	13.553	0.000	0.464	0.566
GS5	0.872	0.039	22.208	0.000	0.872	0.819
GS6	1.018	0.046	21.914	0.000	1.018	0.811
GS7	0.834	0.039	21.205	0.000	0.834	0.794
GS8	0.570	0.035	16.479	0.000	0.570	0.662

Variances:

	Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
.GS1	0.502	0.036	13.849	0.000	0.502	0.402
.GS2	0.971	0.063	15.357	0.000	0.971	0.658
.GS3	0.527	0.039	13.627	0.000	0.527	0.381
.GS4	0.458	0.030	15.431	0.000	0.458	0.680
.GS5	0.374	0.029	12.990	0.000	0.374	0.330
.GS6	0.538	0.041	13.153	0.000	0.538	0.342
.GS7	0.408	0.030	13.505	0.000	0.408	0.370
.GS8	0.417	0.028	14.954	0.000	0.417	0.562
f1	1.000				1.000	1.000

Unidimensional Model with correlated residual

```
m2='
f1=~GS1+GS2+GS3+GS4+GS5+GS6+GS7+GS8
GS4~~GS8
'
fitm2=cfa(m2,data=ont2,std.lv=T)
summary(fitm2,fit.measures=T,standardized=T)
```

lavaan 0.6-3 ended normally after 20 iterations

Optimization method	NLMINB
Number of free parameters	17
Number of observations	522
Estimator	ML
Model Fit Test Statistic	181.997
Degrees of freedom	19
P-value (Chi-square)	0.000

Model test baseline model:

Minimum Function Test Statistic	2411.252
Degrees of freedom	28
P-value	0.000

User model versus baseline model:

Comparative Fit Index (CFI)	0.932
Tucker-Lewis Index (TLI)	0.899

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-5051.452
Loglikelihood unrestricted model (H1)	-4960.454
Number of free parameters	17
Akaike (AIC)	10136.905
Bayesian (BIC)	10209.285
Sample-size adjusted Bayesian (BIC)	10155.323

Root Mean Square Error of Approximation:

RMSEA	0.128
90 Percent Confidence Interval	0.112 0.146
P-value RMSEA <= 0.05	0.000

Standardized Root Mean Square Residual:

SRMR	0.061

Parameter Estimates:

```
  Information                 Expected
  Information saturated (h1) model    Structured
  Standard Errors             Standard

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  f1 =~
    GS1               0.879    0.042   20.913    0.000    0.879    0.787
    GS2               0.694    0.051   13.697    0.000    0.694    0.571
    GS3               0.948    0.044   21.632    0.000    0.948    0.805
    GS4               0.426    0.035   12.197    0.000    0.426    0.518
    GS5               0.885    0.039   22.694    0.000    0.885    0.831
    GS6               1.022    0.046   22.022    0.000    1.022    0.815
    GS7               0.815    0.040   20.470    0.000    0.815    0.776
    GS8               0.537    0.035   15.260    0.000    0.537    0.624

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
 .GS4 ~~
   .GS8               0.244    0.025    9.854    0.000    0.244    0.515

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
    .GS1              0.475    0.035   13.573    0.000    0.475    0.381
    .GS2              0.994    0.065   15.396    0.000    0.994    0.674
    .GS3              0.488    0.037   13.226    0.000    0.488    0.352
    .GS4              0.493    0.032   15.561    0.000    0.493    0.731
    .GS5              0.351    0.028   12.604    0.000    0.351    0.310
    .GS6              0.530    0.041   13.015    0.000    0.530    0.336
    .GS7              0.440    0.032   13.762    0.000    0.440    0.399
    .GS8              0.454    0.030   15.151    0.000    0.454    0.611
     f1               1.000                      1.000    1.000
```

Two factor congeneric model

```
m4='
 g1=~GS1+GS3+GS5+GS6
 g2=~GS2+GS4+GS7+GS8
 '
 fitm4=cfa(m4,data=ont2,std.lv=T)
 summary(fitm4,fit.measures=T,standardized=T)
```

lavaan 0.6-3 ended normally after 21 iterations

Optimization method	NLMINB
Number of free parameters	17
Number of observations	522
Estimator	ML
Model Fit Test Statistic	124.674
Degrees of freedom	19
P-value (Chi-square)	0.000

Model test baseline model:

Minimum Function Test Statistic	2411.252
Degrees of freedom	28
P-value	0.000

User model versus baseline model:

Comparative Fit Index (CFI)	0.956
Tucker-Lewis Index (TLI)	0.935

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-5022.791
Loglikelihood unrestricted model (H1)	-4960.454
Number of free parameters	17
Akaike (AIC)	10079.581
Bayesian (BIC)	10151.962
Sample-size adjusted Bayesian (BIC)	10098.000

Root Mean Square Error of Approximation:

RMSEA	0.103
90 Percent Confidence Interval	0.086 0.121
P-value RMSEA <= 0.05	0.000

Standardized Root Mean Square Residual:

SRMR	0.050

Parameter Estimates:

Information	Expected
Information saturated (h1) model	Structured
Standard Errors	Standard

Latent Variables:

	Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
g1 =~						
GS1	0.894	0.042	21.347	0.000	0.894	0.800
GS3	0.979	0.043	22.612	0.000	0.979	0.832
GS5	0.899	0.039	23.155	0.000	0.899	0.845
GS6	1.015	0.047	21.672	0.000	1.015	0.809
g2 =~						
GS2	0.743	0.051	14.602	0.000	0.743	0.612
GS4	0.567	0.033	17.062	0.000	0.567	0.691
GS7	0.894	0.039	22.808	0.000	0.894	0.851
GS8	0.688	0.033	20.830	0.000	0.688	0.799

Covariances:

	Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
g1 ~~						
g2	0.778	0.024	31.974	0.000	0.778	0.778

Variances:

	Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
.GS1	0.449	0.034	13.027	0.000	0.449	0.359
.GS3	0.427	0.035	12.200	0.000	0.427	0.308
.GS5	0.325	0.028	11.764	0.000	0.325	0.287
.GS6	0.545	0.042	12.837	0.000	0.545	0.346
.GS2	0.923	0.063	14.764	0.000	0.923	0.626
.GS4	0.352	0.025	14.007	0.000	0.352	0.523
.GS7	0.305	0.031	9.879	0.000	0.305	0.276
.GS8	0.268	0.023	11.871	0.000	0.268	0.362
g1	1.000			1.000	1.000	
g2	1.000			1.000	1.000	

Two factor noncongeneric model

fa(ont2,2)

Factor Analysis using method = minres

Call: fa(r = ont2, nfactors = 2)
Standardized loadings (pattern matrix) based upon correlation matrix

	MR1	MR2	h2	u2	com
GS1	0.82	-0.01	0.65	0.35	1.0
GS2	0.28	0.40	0.37	0.63	1.8
GS3	0.91	-0.09	0.73	0.27	1.0
GS4	-0.05	0.79	0.58	0.42	1.0
GS5	0.80	0.05	0.70	0.30	1.0
GS6	0.69	0.17	0.65	0.35	1.1
GS7	0.41	0.50	0.67	0.33	1.9
GS8	0.00	0.87	0.76	0.24	1.0

	MR1	MR2
SS loadings	3.07	2.04
Proportion Var	0.38	0.26
Cumulative Var	0.38	0.64
Proportion Explained	0.60	0.40
Cumulative Proportion	0.60	1.00

With factor correlations of

	MR1	MR2
MR1	1.00	0.62
MR2	0.62	1.00

Mean item complexity = 1.2
Test of the hypothesis that 2 factors are sufficient.

The degrees of freedom for the null model are 28 and the objective function was 4.62 with Chi Square of 2390.47
The degrees of freedom for the model are 13 and the objective function was 0.03

The root mean square of the residuals (RMSR) is 0.01
The df corrected root mean square of the residuals is 0.02

The harmonic number of observations is 522 with the empirical chi square 4.78 with prob < 0.98
The total number of observations was 522 with Likelihood Chi Square = 17.9 with prob < 0.16

Tucker Lewis Index of factoring reliability = 0.996
RMSEA index = 0.027 and the 90 % confidence intervals are 0 0.055
BIC = -63.45
Fit based upon off diagonal values = 1
Measures of factor score adequacy

	MR1	MR2
Correlation of (regression) scores with factors	0.95	0.93
Multiple R square of scores with factors	0.91	0.86

Minimum correlation of possible factor scores 0.81 0.73

##Partial Invariance Model for Gender

```
 o3<-cfa(metric,data=ont41,group="sex1",
 group.equal=c("loadings","intercepts"),group.partial =
c("f1=~GS8","f1=~GS4","GS8~1","GS4~1"),missing='ml')
 summary(o3,fit.measures=T)
```

lavaan 0.6-3 ended normally after 51 iterations

Optimization method	NLMINB
Number of free parameters	52
Number of equality constraints	12

Number of observations per group	
1	262
0	260
Number of missing patterns per group	
1	1
0	1

Estimator	ML
Model Fit Test Statistic	217.500
Degrees of freedom	48
P-value (Chi-square)	0.000

Chi-square for each group:

1	108.764
0	108.736

Model test baseline model:

Minimum Function Test Statistic	2454.407
Degrees of freedom	56
P-value	0.000

User model versus baseline model:

Comparative Fit Index (CFI)	0.929
Tucker-Lewis Index (TLI)	0.918

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-5024.167
Loglikelihood unrestricted model (H1)	-4915.417
Number of free parameters	40
Akaike (AIC)	10128.334
Bayesian (BIC)	10298.641
Sample-size adjusted Bayesian (BIC)	10171.672

Root Mean Square Error of Approximation:

RMSEA	0.116
90 Percent Confidence Interval	0.101 0.132
P-value RMSEA <= 0.05	0.000

Standardized Root Mean Square Residual:

SRMR	0.067

Parameter Estimates:

Information	Observed
Observed information based on	Hessian
Standard Errors	Standard

Group 1 [1]:

Latent Variables:

		Estimate	Std.Err	z-value	P(>|z|)
f1 =~					
GS1	(.p1.)	0.898	0.053	16.939	0.000
GS2	(.p2.)	0.729	0.058	12.587	0.000
GS3	(.p3.)	0.966	0.056	17.242	0.000
GS4		0.350	0.044	7.925	0.000
GS5	(.p5.)	0.904	0.050	18.006	0.000
GS6	(.p6.)	1.046	0.059	17.584	0.000
GS7	(.p7.)	0.835	0.049	16.881	0.000
GS8		0.456	0.047	9.700	0.000

Covariances:

	Estimate	Std.Err	z-value	P(>|z|)
.GS4 ~~				

.GS8		0.193	0.030	6.459	0.000

Intercepts:

		Estimate	Std.Err	z-value	P(>|z|)
f1		0.000			
.GS1	(.20.)	3.053	0.064	47.338	0.000
.GS2	(.21.)	3.055	0.064	48.095	0.000
.GS3	(.22.)	3.004	0.069	43.760	0.000
.GS4		4.431	0.045	98.737	0.000
.GS5	(.24.)	3.282	0.063	52.272	0.000
.GS6	(.25.)	3.416	0.074	46.435	0.000
.GS7	(.26.)	3.697	0.060	61.416	0.000
.GS8		4.130	0.049	84.084	0.000

Variances:

	Estimate	Std.Err	z-value	P(>|z|)
f1	1.000			
.GS1	0.483	0.051	9.473	0.000
.GS2	1.129	0.104	10.900	0.000
.GS3	0.542	0.058	9.400	0.000
.GS4	0.405	0.037	11.073	0.000
.GS5	0.346	0.039	8.831	0.000
.GS6	0.553	0.060	9.245	0.000
.GS7	0.419	0.044	9.482	0.000
.GS8	0.424	0.039	10.812	0.000

Group 2 [0]:

Latent Variables:

		Estimate	Std.Err	z-value	P(>|z|)
f1 =~					
GS1	(.p1.)	0.898	0.053	16.939	0.000
GS2	(.p2.)	0.729	0.058	12.587	0.000
GS3	(.p3.)	0.966	0.056	17.242	0.000
GS4		0.550	0.057	9.611	0.000
GS5	(.p5.)	0.904	0.050	18.006	0.000
GS6	(.p6.)	1.046	0.059	17.584	0.000
GS7	(.p7.)	0.835	0.049	16.881	0.000
GS8		0.665	0.058	11.422	0.000

Covariances:

	Estimate	Std.Err	z-value	P(>|z|)
.GS4 ~~				
.GS8	0.244	0.037	6.631	0.000

Intercepts:

	Estimate	Std.Err	z-value	P(>|z|)
f1	0.110	0.090	1.218	0.223
.GS1	(.20.)	3.053	0.064	47.338	0.000
.GS2	(.21.)	3.055	0.064	48.095	0.000
.GS3	(.22.)	3.004	0.069	43.760	0.000
.GS4	4.090	0.058	70.471	0.000
.GS5	(.24.)	3.282	0.063	52.272	0.000
.GS6	(.25.)	3.416	0.074	46.435	0.000
.GS7	(.26.)	3.697	0.060	61.416	0.000
.GS8	3.935	0.061	64.104	0.000

Variances:

	Estimate	Std.Err	z-value	P(>|z|)
f1	0.905	0.124	7.305	0.000
.GS1	0.470	0.048	9.721	0.000
.GS2	0.850	0.080	10.573	0.000
.GS3	0.442	0.048	9.240	0.000
.GS4	0.507	0.047	10.694	0.000
.GS5	0.360	0.039	9.171	0.000
.GS6	0.503	0.054	9.331	0.000
.GS7	0.454	0.047	9.676	0.000
.GS8	0.445	0.044	10.214	0.000

##Scalar Invariance model for Education

```
edu_o3<-cfa(metric,data=ont42,group="edu",
group.equal=c("loadings","intercepts"),missing='ml')
summary(edu_o3,fit.measures=T)
```

lavaan 0.6-3 ended normally after 52 iterations

Optimization method	NLMINB
Number of free parameters	52
Number of equality constraints	16
Number of observations per group	
2	290
3	192
Number of missing patterns per group	
2	1
3	1

Estimator	ML
Model Fit Test Statistic	203.153
Degrees of freedom	52
P-value (Chi-square)	0.000

Chi-square for each group:

2	128.859
3	74.294

Model test baseline model:

Minimum Function Test Statistic	2308.351
Degrees of freedom	56
P-value	0.000

User model versus baseline model:

Comparative Fit Index (CFI)	0.933
Tucker-Lewis Index (TLI)	0.928

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-4664.876
Loglikelihood unrestricted model (H1)	-4563.299
Number of free parameters	36
Akaike (AIC)	9401.752
Bayesian (BIC)	9552.158
Sample-size adjusted Bayesian (BIC)	9437.897

Root Mean Square Error of Approximation:

RMSEA	0.110
90 Percent Confidence Interval	0.094 0.126
P-value RMSEA <= 0.05	0.000

Standardized Root Mean Square Residual:

SRMR	0.061

Parameter Estimates:

Information	Observed

Observed information based on Hessian
Standard Errors Standard

Group 1 [2]:

Latent Variables:

		Estimate	Std.Err	z-value	P(>|z|)
f1 =~					
GS1	(.p1.)	0.902	0.051	17.617	0.000
GS2	(.p2.)	0.694	0.057	12.205	0.000
GS3	(.p3.)	0.963	0.054	17.834	0.000
GS4	(.p4.)	0.446	0.039	11.377	0.000
GS5	(.p5.)	0.907	0.049	18.513	0.000
GS6	(.p6.)	1.054	0.058	18.280	0.000
GS7	(.p7.)	0.838	0.049	17.197	0.000
GS8	(.p8.)	0.554	0.041	13.614	0.000

Covariances:

		Estimate	Std.Err	z-value	P(>|z|)
.GS4 ~~					
.GS8		0.210	0.033	6.406	0.000

Intercepts:

		Estimate	Std.Err	z-value	P(>|z|)
f1		0.000			
.GS1	(.20.)	3.056	0.062	49.295	0.000
.GS2	(.21.)	3.034	0.062	49.234	0.000
.GS3	(.22.)	3.005	0.066	45.551	0.000
.GS4	(.23.)	4.262	0.042	102.556	0.000
.GS5	(.24.)	3.274	0.061	53.876	0.000
.GS6	(.25.)	3.395	0.071	47.679	0.000
.GS7	(.26.)	3.685	0.059	62.927	0.000
.GS8	(.27.)	4.021	0.045	89.127	0.000

Variances:

	Estimate	Std.Err	z-value	P(>|z|)
f1	1.000			
.GS1	0.426	0.044	9.779	0.000
.GS2	1.040	0.090	11.514	0.000
.GS3	0.492	0.050	9.907	0.000
.GS4	0.506	0.044	11.556	0.000
.GS5	0.373	0.039	9.620	0.000
.GS6	0.517	0.053	9.706	0.000
.GS7	0.457	0.045	10.189	0.000

.GS8		0.443	0.040	11.095	0.000

Group 2 [3]:

Latent Variables:

		Estimate	Std.Err	z-value	P(>|z|)
f1 =~					
GS1	(.p1.)	0.902	0.051	17.617	0.000
GS2	(.p2.)	0.694	0.057	12.205	0.000
GS3	(.p3.)	0.963	0.054	17.834	0.000
GS4	(.p4.)	0.446	0.039	11.377	0.000
GS5	(.p5.)	0.907	0.049	18.513	0.000
GS6	(.p6.)	1.054	0.058	18.280	0.000
GS7	(.p7.)	0.838	0.049	17.197	0.000
GS8	(.p8.)	0.554	0.041	13.614	0.000

Covariances:

		Estimate	Std.Err	z-value	P(>|z|)
.GS4 ~~					
.GS8		0.318	0.045	7.098	0.000

Intercepts:

		Estimate	Std.Err	z-value	P(>|z|)
f1		0.113	0.096	1.181	0.238
.GS1	(.20.)	3.056	0.062	49.295	0.000
.GS2	(.21.)	3.034	0.062	49.234	0.000
.GS3	(.22.)	3.005	0.066	45.551	0.000
.GS4	(.23.)	4.262	0.042	102.556	0.000
.GS5	(.24.)	3.274	0.061	53.876	0.000
.GS6	(.25.)	3.395	0.071	47.679	0.000
.GS7	(.26.)	3.685	0.059	62.927	0.000
.GS8	(.27.)	4.021	0.045	89.127	0.000

Variances:

	Estimate	Std.Err	z-value	P(>|z|)
f1	0.936	0.135	6.924	0.000
.GS1	0.519	0.062	8.370	0.000
.GS2	0.918	0.099	9.296	0.000
.GS3	0.510	0.063	8.036	0.000
.GS4	0.495	0.053	9.396	0.000
.GS5	0.326	0.043	7.530	0.000
.GS6	0.561	0.070	8.019	0.000
.GS7	0.405	0.050	8.129	0.000
.GS8	0.484	0.053	9.198	0.000

```
## IRT item parameters

coef(n2,IRTpars=TRUE)
$GS1
      a     b1     b2    b3    b4
par 2.461 -1.825 -0.561 0.283 1.529

$GS2
      a     b1     b2    b3    b4
par 1.465 -2.049 -0.533 0.232 1.674

$GS3
      a     b1    b2    b3    b4
par 2.559 -1.619 -0.47 0.277 1.402

$GS4
      a     b1     b2     b3    b4
par 1.515 -4.797 -2.762 -1.465 0.039

$GS5
      a     b1     b2    b3    b4
par 3.074 -2.008 -0.829 0.016 1.239

$GS6
      a     b1     b2     b3    b4
par 3.125 -1.554 -0.763 -0.223 0.781

$GS7
      a     b1     b2     b3    b4
par 2.899 -2.382 -1.189 -0.416 0.687

$GS8
      a     b1     b2     b3    b4
par 1.922 -4.069 -2.135 -1.003 0.498

$GroupPars
   MEAN_1 COV_11
par      0      1


##ECV
```

```
n1=c(cor(dat1[,126],dat1[,c(80:85,172:175)]))
n2=c(cor(dat1[,27],dat1[,c(80:85,172:175)]))

# # cor(dat1[,87],dat1[,126])
# n1=c(cor(dat1[,126],dat1[,c(80:85,172:175)]))
# n2=c(cor(dat1[,87],dat1[,c(80:85,172:175)]))

#Steiger 1980's formula
dep_cor<-function(cor1,cor2,control,n){
        detR=1-cor1^2-cor2^2-control^2+2*cor1*cor2*control
        rbar=(cor1+cor2)/2
        num=(cor1-cor2)*sqrt((n-1)*(1+control))
        den=sqrt(2*((n-1)/(n-3))*detR+rbar^2*(1-control)^3)
        df=n-3
        t1=num/den
        rep1=matrix(NA,nrow=1,ncol=2)
        colnames(rep1)=c('tvalue','df')
        rep1[1,]<-c(t1,df)
        return(rep1)
}

dep_cor(.4,.5,.1,103)

p1=matrix(NA,nrow=length(n1),ncol=2)
for(i in 1:length(n1)){
        p2=dep_cor(n1[i],n2[i],.728,522)
        p1[i,]=p2
}
```

```
      tvalues  df
[1,] -0.4422268 519
[2,]  1.1231446 519
[3,]  0.5577912 519
[4,] -0.3024296 519
```